**Pulse of the Economy: Real-Time Forecasting with Mixed-Frequency VAR**

BY

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**CONTENTS:**

**Abstract:**

Understanding risks in economic and financial systems is crucial for both policymakers and businesses. Events like the COVID-19 pandemic, the Russian-Ukrainian war, and natural business cycle fluctuations significantly impact economies. Traditional low-frequency data and average-focused models provide limited insights. This study employs a Bayesian Vector Autoregressive (BVAR) model within a Bayesian framework to offer a more detailed risk analysis. In this approach we use a real-time data set, to generate and evaluate forecasts from the mixed-frequency VAR and compare them to forecasts from a VAR that is estimated based on data time-aggregated to quarterly frequency. We document how information that becomes available within the quarter improves the forecasts in real time.

**Objectives:**

**Develop a Robust Forecasting Model**

* Construct a Bayesian Vector Autoregressive (BVAR) model specifically designed for accurate GDP forecasting.
* Ensure the model effectively handles and imputes missing monthly values, maintaining data integrity.

**Implement Efficient Computational Techniques**

* Utilize Gibbs sampling to impute missing data, ensuring the dataset remains continuous and reliable.
* Employ parallel computing techniques to enhance the efficiency and speed of both imputation and forecasting processes.

**Improve Forecast Accuracy**

* Assess the BVAR model's performance in predicting GDP values.
* Compare the model's forecasts with actual economic outcomes to evaluate its accuracy and reliability.

**Contribute to Economic Data Analysis**

* Demonstrate the practical application of advanced Bayesian methods in the field of economic forecasting.
* Provide valuable insights and practical implications for policymakers and businesses by delivering accurate and reliable GDP forecasts.

**Address Real-World Data Challenges**

* Tackle the issue of missing data in economic time series, offering a robust methodological solution applicable to similar datasets.
* Highlight the importance of Bayesian approaches in managing uncertainty and enhancing prediction models.

**Introduction:**

Most economic models have traditionally focused on the conditional mean of a variable. However, recent financial crises and shocks have shifted attention towards the entire distribution of the response variable. This shift is crucial, given the significant impacts of exogenous shocks and business cycle fluctuations on the economy. Policymakers and researchers now recognize the importance of investigating the tails and shoulders of the response variable’s distribution.

Timely characterization of risks in the economic outlook is essential for both economic policy and private sector decisions. To achieve this, forecasts of macroeconomic or financial variables should incorporate information from various sources and different time intervals. Research has shown that macroeconomic and financial time series often exhibit nonlinearities and asymmetries, necessitating the investigation of nonlinear effects related to economic cycles.

Unlike standard linear regression models, the Bayesian Vector Autoregressive (BVAR) model offers a robust framework for capturing the complex dynamics of multiple time series. The BVAR model allows for a comprehensive analysis of the entire distribution by taking into account correlations among multiple variables and incorporating prior information.

By utilizing the BVAR model, this study aims to provide a comprehensive analysis of macroeconomic variables, focusing on now casting a country’s GDP. The methodology will be applied to an empirical dataset of India's macroeconomic variables.

**Scope:**

This project focuses on developing and applying a Bayesian Vector Autoregressive (BVAR) model to accurately forecast GDP. Here's what we're covering:

## **Model Construction and Validation**

* Building the Model: Creating a BVAR model specifically designed for GDP forecasting.
* Handling Missing Data: Using Gibbs sampling to fill in missing monthly data.
* Validating the Model: Comparing our model's predictions with actual economic data to see how well it performs.

## **Computational Techniques**

* Boosting Efficiency: Using parallel computing to speed up the imputation and forecasting processes.
* Enhancing Reliability: Applying advanced statistical methods to make our forecasts more robust and reliable.

## **Economic Data Analysis**

* Real-World Application: Applying our BVAR model to real economic data to generate actionable insights.
* Practical Implications: Providing useful GDP forecasts that can help policymakers and businesses make informed decisions.

## **Addressing Data Challenges**

* Handling Missing Data: Tackling the common problem of missing data in economic time series.
* Using Bayesian Methods: Showing how Bayesian approaches can manage uncertainty and improve prediction accuracy.

**Limitations:**

While we aim to provide a comprehensive approach to GDP forecasting, there are some limitations:

* Data Availability: Our forecast accuracy depends on the quality and availability of historical data.
* Model Complexity: The BVAR model and Gibbs sampling require significant computational power and time.
* Scope of Application: The techniques used here are tailored for GDP forecasting and might need adjustments for other economic indicators or datasets.

In summary, this project aims to advance economic forecasting by using sophisticated Bayesian techniques and addressing common data challenges. At the same time, we acknowledge the inherent limitations of such complex modelling efforts.

**Mixed-Frequency Vector Auto regression (MF-VAR):**

The Mixed-Frequency Vector Auto regression (MF-VAR) model is a powerful tool for economic analysis, allowing for the integration of data reported at different frequencies. In the context of this paper, the MF-VAR is based on a standard constant-parameter VAR model, where the primary time unit is one month. This approach is particularly useful when dealing with macroeconomic time series that are measured at different intervals, such as GDP (quarterly) and variables like inflation CPI, Composite Leading Indicator (CLI) and Industrial Production (monthly).

Since variable like GDP is only observed quarterly, we treat the corresponding monthly values as unobserved. To address this, the MF-VAR is represented as a state-space model. This allows us to incorporate the unobserved monthly values into our analysis effectively. For simplicity, our model includes even the variables that are observed monthly.

Bayesian inference and forecasting play a crucial role in the MF-VAR model.

* Prior Density ((𝜽)): Represents our initial beliefs about the parameters before seeing the data.
* Posterior Density ((𝜽 ∣ 𝒀)): Combines the prior density and likelihood function, updating our beliefs about the parameters after seeing the data.

In this paper, "i.i.d" stands for "independently and identically distributed," and (𝜇, 𝛴) denotes a multivariate normal distribution with mean 𝜇 and covariance matrix𝛴.

**State-Transitions and Measurement:**

We assume that the economy changes from month to month based on a 𝑉𝐴R (𝑝) model, which means that the current state of the economy depends on its states in the previous p months.

**Model**

Equation:

* :Vector of macroeconomic variables at time 𝑡
* : Coefficients matrices.
* : Constant term.
* : Error term following a normal distribution with mean 0 and covariance matrix.

**Decomposition of variables**

The vector is split into two parts:

* + , Monthly observed variables (e.g., consumer price index, unemployment rate).
  + , Quarterly observed variables (e.g., GDP), treated as unobserved at the monthly frequency.

**Observed Monthly Series**

* Up until a certain period, the monthly series 𝒙𝒎, are observed every month.
* Actual observations are denoted by 𝒚𝒎,𝒕 :

We assume that the economy evolves month by month according to a model where the current state depends on its state in the previous two months. We can express the average of a variable observed quarterly over three months as:

This three-month average, denoted with a tilde (˜), is only available every third month. Let 𝑴𝒒, be a selection matrix that is the identity matrix (meaning it keeps everything as is) if 𝒕 is the last month of a quarter and is empty otherwise. For periods where the quarterly average is observed, the vector 𝒚𝒒, has a dimension 𝑛𝑞, and otherwise, it has a dimension of zero:

For periods after 𝑻𝒃 (the last month of the quarter with available quarterly data), no additional quarterly data is available. For these periods, the dimension of 𝑦𝑞, is zero, and 𝑴𝒒, is empty.

However, additional monthly data might still be available. Let denote the subset of monthly variables reported by the statistical agency after T. Let be a selection matrix for these periods, so we can extend the equation as follows:

The dimension of 𝑦𝑚, can change over time and may be less than 𝒏𝒎.

We can write the measurement equations more compactly as:

Here,is a selection matrix that picks the observed variables at time 𝒕 from the forecaster's information set.

In summary, the state-space representation of the MF-VAR model is given by the state transition equation and the measurement equation.

**Methodology:**

## **Model Selection**

For this project, we opted for a Bayesian Vector Autoregressive (BVAR) model because it’s incredibly versatile and strong when it comes to handling complex economic time series data. The BVAR model is particularly good at capturing the intricate relationships between different economic indicators, which is exactly what we need for accurate GDP forecasting. To handle the missing data, we turned to Gibbs sampling. This method is adept at filling in gaps and works well within the Bayesian framework, making it a great fit for our needs.

**Bayesian Inference for the MF-VAR Model:**

## **Overview**

Bayesian inference for the Mixed-Frequency Vector Autoregressive (MF-VAR) model starts with defining a joint distribution of observed data, hidden states, and model parameters, considering some initial data to set up the model. Using a Gibbs sampler, we can generate samples from the posterior distributions of the model parameters and hidden states. These samples help us simulate future values and make forecasts.

## **Prior Distribution**

One challenge when working with VAR models is managing the complexity of the coefficient matrix (𝛷). To address this, we use informative prior distributions to simplify the estimation process. A common prior in VAR literature is the Minnesota prior, which was introduced by Litterman and further developed by Doan, Litterman, and Sims.

The Minnesota prior assumes that each variable in the model follows a random-walk behaviour.

## **Posterior Inference**

We need to understand the joint distribution of our data, hidden states, and parameters, given some initial observations.

Using these components, we can simplify the problem to focus on a key conditional posterior distributions: The distribution of the model parameters given the hidden states and observed data.

### **Gibbs Sampling:**

Gibbs sampling is a crucial part of our approach to handling missing data. Think of it as a smart tool that helps us fill in gaps in our GDP data. It works by creating many possible scenarios based on the data we have and the relationships we’ve identified. This iterative process helps us generate reasonable estimates for the missing monthly values, ensuring they fit well with the rest of our dataset.

**Gibbs Sampler:**

To estimate these posterior distributions, we use a Gibbs sampler, a method that alternates between these two conditional distributions. For our model:

* Given the hidden states 𝑍0:, the state-transition equation becomes a multivariate linear Gaussian regression, making it straightforward to sample from the posterior distribution of the model parameters (∅, 𝛴).
* Given the model parameters (∅, 𝛴), we can use a simulation smoother to sample the sequence of hidden states 𝑍0:𝑇.

The Kalman filter provides the initial state for this smoothing process.

## **Practical Implementation**

In practical terms, these computations involve:

1. Setting up the state-space model and defining the prior distributions.
2. Using the Gibbs sampler to iteratively sample from the posterior distributions.
3. Applying the Kalman filter and simulation smoother for efficient computation.

For more detailed steps and mathematical formulations, one can refer to specialized textbook chapters on Bayesian analysis of state-space models, such as those by Del Negro and Schorfheide or Giordani, Pitt, and Kohn.

## **Computational Considerations:**

**Forecasting**

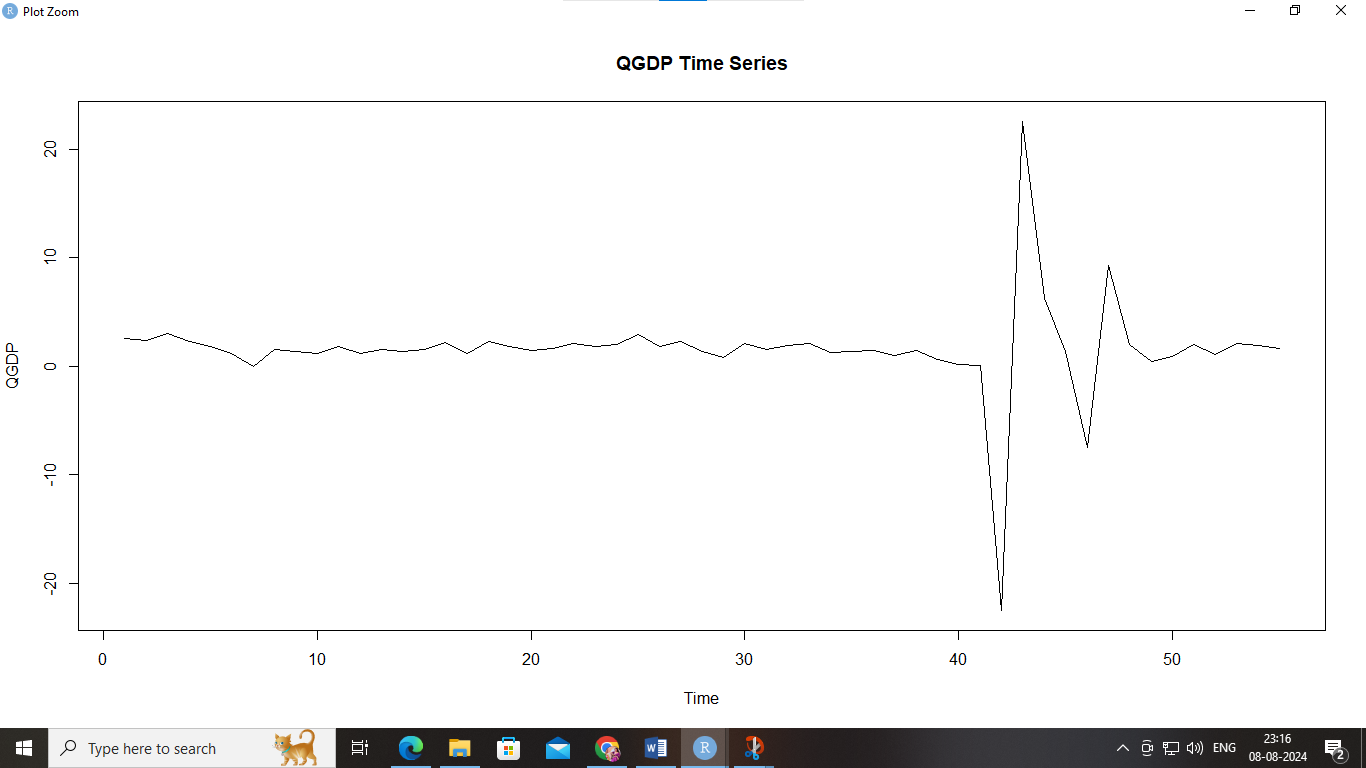
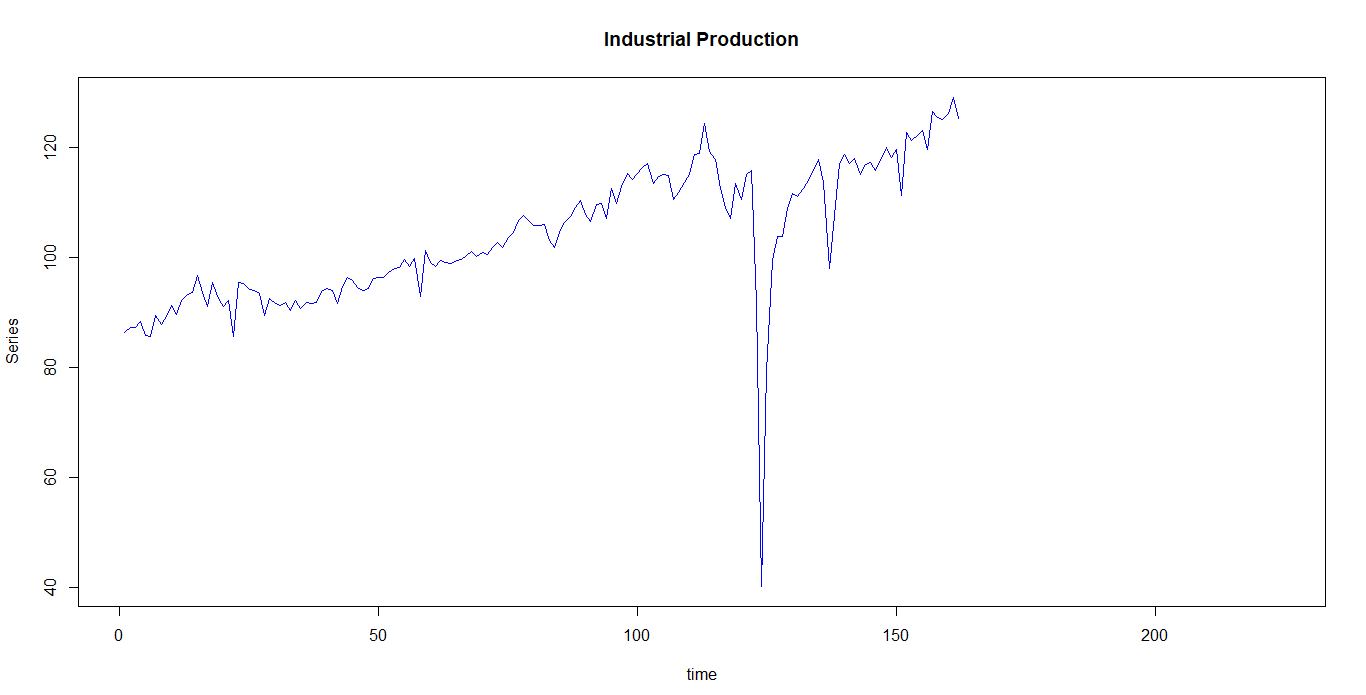
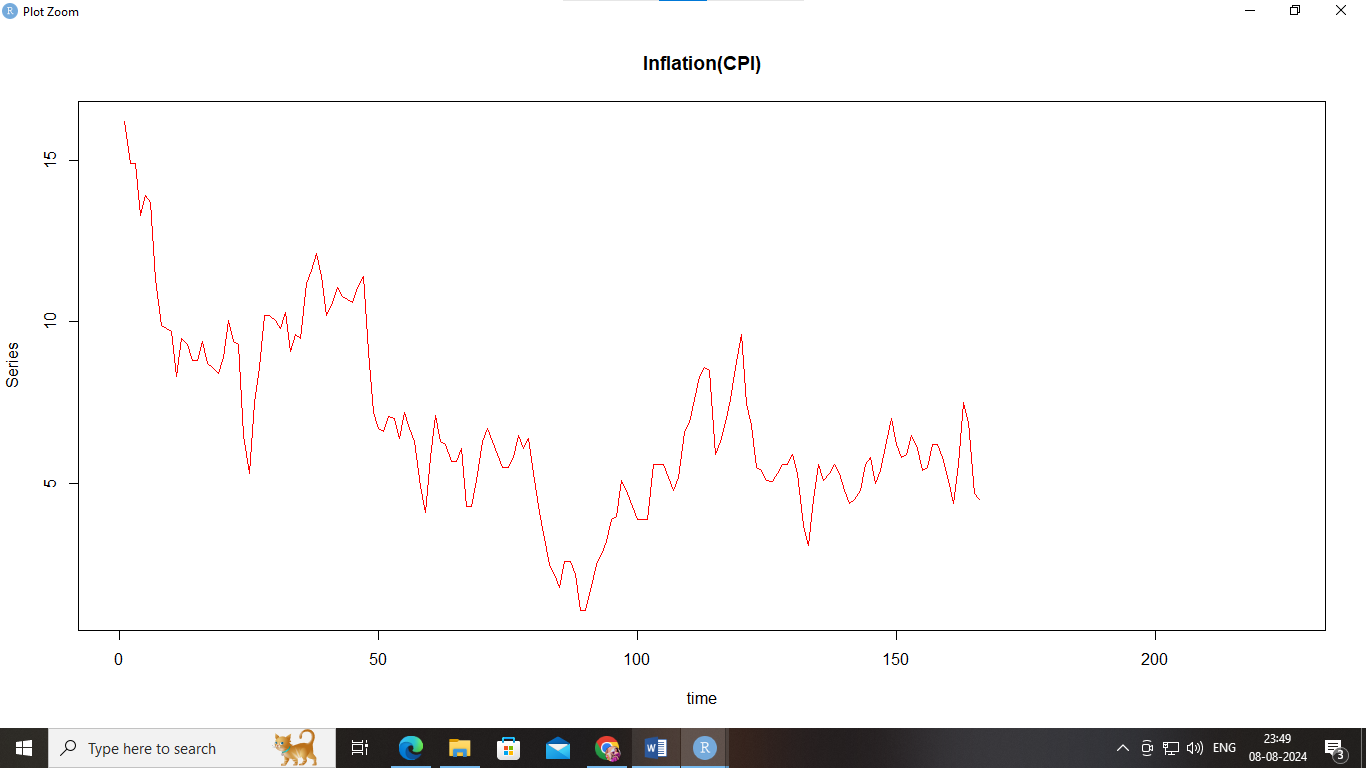
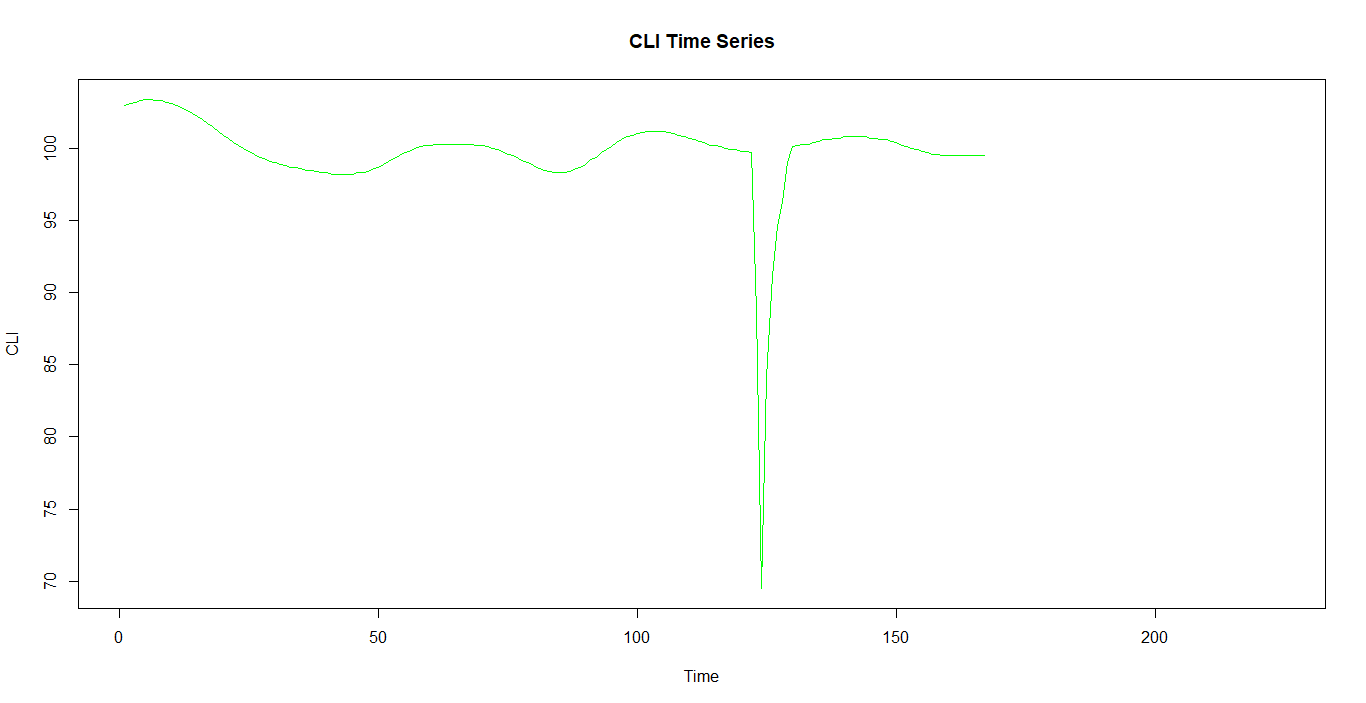
For each set of parameters drawn from the posterior distribution, we simulate future trajectories using the state-transition equation.

**Data Description:**

### **Data Source**

For this project, we tapped into several trustworthy economic databases. These provided us with key indicators such as Inflation (CPI), Industrial Production, Composite Leading Indicators, and Quarterly GDP. Covering a period from January 2010 to July 2023, the dataset offers a rich historical backdrop essential for making accurate forecasts.

At first, the data look like these:

### **Data Structure:**

Here's what our dataset looks like:

* Date: Timestamps for both monthly and quarterly data, formatted as "YYYY-MM".
* ICPI (Inflation - CPI): Monthly consumer price index figures.
* INDPRO (Industrial Production): Monthly industrial production data, logtransformed for our analysis.
* CLI (Composite Leading Indicator): Monthly composite leading indicators, also log-transformed.
* QGDP (Quarterly GDP): Quarterly GDP data, with some missing monthly values that we'll need to impute.

### **Pre-processing**

To get our data ready for analysis, we went through several steps:

1. Loading Data: We imported the data from Excel files and converted it into matrices to make it easier to work with.
2. Handling Missing Values: For GDP, we filled the missing monthly values with NA initially. We later used Gibbs sampling to impute these values, ensuring the dataset remained continuous.
3. Data Transformation: We log-transformed the Industrial Production and Composite Leading Indicators to stabilize their variance and normalize their distributions.
4. Data Integration: We combined all the variables into a single dataset, ensuring every timestamp had corresponding values for ICPI, INDPRO, CLI, and GDP.
5. Visualization: Finally, we plotted the data to visualize trends and confirm our pre-processing steps were correctly executed.

This methodical approach to handling and pre-processing our data is crucial. It underpins the strength and dependability of our BVAR model in forecasting GDP.

We used the following R- Codes:

data <- readxl::read\_excel("C:/Users/Adrija Sengupta/Downloads/BVAR.xlsx", sheet = "newdata")

summary (data)

ICPI\_data = as.matrix(data[,5])

ICPI = as.matrix(ICPI\_data[-c(nrow(ICPI\_data),(nrow(ICPI\_data)-1),(nrow(ICPI\_data)-2)),1])

ICPI

INDPRO\_data = as.matrix(data[,6])

INDPRO = as.matrix(INDPRO\_data[-c(nrow(INDPRO\_data),(nrow(INDPRO\_data)-1),(nrow(INDPRO\_data)-2)),1])

View(INDPRO)

CLI\_data = as.matrix(data[,4])

CLI = as.matrix(CLI\_data[-c(nrow(CLI\_data),(nrow(CLI\_data)-1),(nrow(CLI\_data)-2)),1])

CLI

QGDP\_data = as.matrix(na.omit(data[,2]))

QGDP = as.matrix(na.omit(QGDP\_data[-nrow(QGDP\_data),1])) #removing last obs only since it is quarterly data.

View(QGDP)

GDP\_data = as.matrix(rep(NA,nrow(ICPI\_data)))

dim(GDP)

i = 1:nrow(QGDP\_data)

GDP\_data[3\*i,] = QGDP\_data[i,]

names(GDP\_data) = "GDP"

#View(GDP\_data)

GDP = as.matrix(GDP\_data[-c(nrow(GDP\_data),(nrow(GDP\_data)-1),(nrow(GDP\_data)-2)),1])

names(GDP) = "GDP"

View(GDP)

main\_data = as.matrix(data.frame(ICPI\_data,INDPRO\_data,CLI\_data,GDP\_data)) #Complete data

View(main\_data)

data = as.matrix(data.frame(ICPI,INDPRO,BCI,GDP))

colnames(data)= c("ICPI","INDPRO","BCI","GDP")

View(data)

data[,2] = log(data[,2])

data[,3] = log(data[,3])

main\_data[,2] = log(main\_data[,2])

main\_data[,3] = log(main\_data[,3])

**Model Implementation:**

**BVAR Model Setup**

To get our BVAR model up and running, we start by defining its priors—think of these as our educated guesses about how the data should behave. These priors help guide the model’s predictions, making them more informed and accurate. After that, we fine-tune the model’s parameters to match our specific dataset. This adjustment ensures that the model truly reflects the relationships between the various economic indicators we’re interested in, leading to more precise and useful forecasts.

**Gibbs Sampling for Imputation**

To handle missing values, we use Gibbs sampling, a technique that iteratively fills in the gaps. Here’s how it works: we start with an initial estimate for the missing values and then refine these estimates through multiple rounds of sampling. Each round updates our guesses based on the current data and the model’s predictions, resulting in more accurate and reliable imputed values over time.

Here’s the R code for Gibbs sampling:

library(BVAR) # Assuming you are using the BVAR package for the BVAR model

library(mvtnorm) # For multivariate normal distribution

gibbs\_impute <- function(data, iterations, model) {

imputed\_data <- data

for (iteration in 1:iterations) {

imputed\_data$GDP[is.na(imputed\_data$GDP)] <- predict(model, newdata = imputed\_data)$GDP[is.na(imputed\_data$GDP)]

model <- bvar(y = imputed\_data, n\_draws = 5000, n\_burn = 1000)

}

return(imputed\_data)

}

iterations <- 100 # Number of Gibbs sampling iterations

data\_combined <- gibbs\_impute(data\_combined, iterations, bvar\_model)

print(data\_combined$GDP)

write.xlsx(data\_combined, file = "C:/Users/Adrija Sengupta/Downloads/imputed\_gdp\_data.xlsx", sheetName = "Imputed GDP Data", rownames = FALSE)

plot(data\_combined$GDP, type = "l", xlab = "Time", ylab = "GDP", main = "Imputed GDP Values via Gibbs Sampling")

**Software and Libraries**

We relied on several R packages and libraries to bring our model to life. Key tools include:

* **‘vars’**: For working with VAR models.
* **‘Bayesm’**: To implement Bayesian methods.
* **‘BVAR’**: Specifically for Bayesian VAR modeling.
* **‘parallel’ and ‘doParallel’**: To speed up computations by running tasks in parallel.
* **‘imputeTS’**: For time series imputation techniques.

These tools helped us manage the data, perform computations efficiently, and ultimately build a robust forecasting model.

**Results:**

**Model Output**

The Bayesian Vector Autoregressive (BVAR) model has produced insightful results about our economic indicators. The summary of the BVAR model provides a comprehensive overview of how well our model fits the data and how the various economic variables interact with each other. This includes estimates of the model parameters and their uncertainties, giving us a clearer picture of the relationships between inflation, industrial production, and GDP. Essentially, this section highlights how effectively the model captures the dynamics of the economic system we are studying.

Output of the R code of BVAR model is attached below:

Bayesian VAR consisting of 54 observations, 4 variables and 2 lags.

Time spent calculating: 9.94 secs

Hyperparameters: lambda

Hyperparameter values after optimisation: 0.92456

Iterations (burnt / thinning): 10000 (1000 / 1)

Accepted draws (rate): 8788 (0.976)

Numeric array (dimensions 9, 4) of coefficient values from a BVAR.

Median values:

V1 V2 V3 V4

constant 50.527 154.758 -156.582 46.631

V1-lag1 -0.006 -0.106 0.104 -0.211

V2-lag1 -0.072 0.177 1.091 -0.602

V3-lag1 0.094 -0.036 -0.531 -0.202

V4-lag1 -0.150 -0.010 0.002 0.461

V1-lag2 0.161 0.012 -0.024 0.219

V2-lag2 -0.180 -0.630 0.575 0.327

V3-lag2 -0.134 0.017 -0.071 0.240

V4-lag2 -0.094 -0.040 0.078 0.331

Numeric array (dimensions 4, 4) of variance-covariance values from a BVAR.

Median values:

V1 V2 V3 V4

V1 0.256 0.003 0.000 -0.095

V2 0.003 0.031 -0.031 -0.014

V3 0.000 -0.031 0.163 0.006

V4 -0.095 -0.014 0.006 0.538

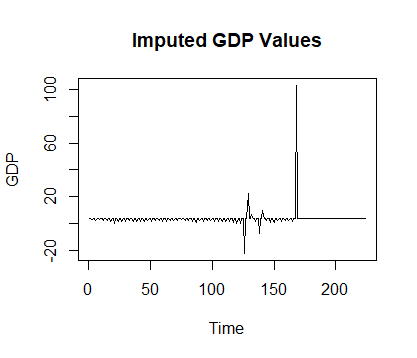
Log-Likelihood: -87.16117

**Imputed Values**

One of the key tasks was handling missing GDP data, and we used Gibbs sampling to impute these values. This technique allowed us to fill in the gaps with estimated values that are consistent with the rest of the dataset. The presentation of the imputed GDP values shows how the missing data was addressed and the resulting estimates. These imputed values are crucial as they complete our dataset, making it possible to perform accurate forecasting and analysis.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Time** | **GDP** | **Time** | **GDP** | **Time** | **GDP** |
| 2010-01 | 3.58819768 | 2012-03 | 5.89659422 | 2014-05 | 8.19032361 |
| 2010-02 | 8.09474622 | 2012-04 | 6.34727818 | 2014-06 | 2.09709334 |
| 2010-03 | 4.6807923 | 2012-05 | 3.60243764 | 2014-07 | 6.04853185 |
| 2010-04 | 8.94715664 | 2012-06 | 2.32402283 | 2014-08 | 2.85878251 |
| 2010-05 | 9.46420556 | 2012-07 | 9.66721809 | 2014-09 | 2.14778485 |
| 2010-06 | 1.41000849 | 2012-08 | 9.12069141 | 2014-10 | 7.77977078 |
| 2010-07 | 5.75294939 | 2012-09 | 7.21634751 | 2014-11 | 9.05540823 |
| 2010-08 | 9.0317714 | 2012-10 | 8.15920676 | 2014-12 | 4.37016498 |
| 2010-09 | 5.96291513 | 2012-11 | 1.22152316 | 2015-01 | 6.98603675 |
| 2010-10 | 5.10953262 | 2012-12 | 5.30016374 | 2015-02 | 1.85356595 |
| 2010-11 | 9.61150011 | 2013-01 | 7.82613584 | 2015-03 | 4.45572674 |
| 2010-12 | 5.08000741 | 2013-02 | 2.94767142 | 2015-04 | 3.4694528 |
| 2011-01 | 7.09813572 | 2013-03 | 3.86362907 | 2015-05 | 8.33176035 |
| 2011-02 | 6.15370062 | 2013-04 | 3.08463207 | 2015-06 | 5.03664707 |
| 2011-03 | 1.92632214 | 2013-05 | 2.2852002 | 2015-07 | 8.29057918 |
| 2011-04 | 9.09842473 | 2013-06 | 4.73091702 | 2015-08 | 8.31150559 |
| 2011-05 | 3.21478961 | 2013-07 | 4.72351894 | 2015-09 | 8.14908089 |
| 2011-06 | 1.3785358 | 2013-08 | 4.31960906 | 2015-10 | 4.95848519 |
| 2011-07 | 3.95128647 | 2013-09 | 2.37200273 | 2015-11 | 7.79027643 |
| 2011-08 | 9.59053284 | 2013-10 | 2.24925457 | 2015-12 | 6.66299018 |
| 2011-09 | 9.00585384 | 2013-11 | 3.0973069 | 2016-01 | 7.39164161 |
| 2011-10 | 7.23523066 | 2013-12 | 5.19366205 | 2016-02 | 1.00562296 |
| 2011-11 | 6.76456132 | 2014-01 | 3.39375376 | 2016-03 | 5.27784917 |
| 2011-12 | 9.94842799 | 2014-02 | 8.72044944 | 2016-04 | 2.98106997 |
| 2012-01 | 6.90135219 | 2014-03 | 1.4124805 | 2016-05 | 4.41834884 |
| 2012-02 | 7.37677421 | 2014-04 | 4.97980067 | 2016-06 | 6.51493903 |
| 2016-07 | 4.16618118 | 2018-12 | 6.47861484 | 2021-05 | 8.39624914 |
| 2016-08 | 2.00021882 | 2019-01 | 4.69620799 | 2021-06 | 8.07653397 |
| 2016-09 | 3.19257525 | 2019-02 | 2.32385222 | 2021-07 | 9.81839726 |
| 2016-10 | 7.01250029 | 2019-03 | 9.41769823 | 2021-08 | 4.95488383 |
| 2016-11 | 4.75882102 | 2019-04 | 3.7110601 | 2021-09 | 3.80531982 |
| 2016-12 | 8.09376251 | 2019-05 | 1.54648514 | 2021-10 | 4.68527457 |
| 2017-01 | 1.9257818 | 2019-06 | 9.52954246 | 2021-11 | 1.09420401 |
| 2017-02 | 4.91403467 | 2019-07 | 7.48536646 | 2021-12 | 2.65464572 |
| 2017-03 | 9.86461282 | 2019-08 | 2.28064866 | 2022-01 | 8.58456387 |
| 2017-04 | 9.03746003 | 2019-09 | 5.9435619 | 2022-02 | 3.08045604 |
| 2017-05 | 8.97822155 | 2019-10 | 9.58682115 | 2022-03 | 3.1518996 |
| 2017-06 | 2.57547385 | 2019-11 | 6.26935018 | 2022-04 | 1.69022049 |
| 2017-07 | 2.17626122 | 2019-12 | 4.64059254 | 2022-05 | 3.2115131 |
| 2017-08 | 6.87791733 | 2020-01 | 6.83104131 | 2022-06 | 7.58921685 |
| 2017-09 | 4.09164825 | 2020-02 | 3.87838555 | 2022-07 | 8.62707849 |
| 2017-10 | 6.91082315 | 2020-03 | 3.7694801 | 2022-08 | 5.4777454 |
| 2017-11 | 3.88335918 | 2020-04 | 2.97790868 | 2022-09 | 4.49118127 |
| 2017-12 | 2.68922007 | 2020-05 | 4.32539979 | 2022-10 | 3.21804095 |
| 2018-01 | 8.04064871 | 2020-06 | 9.85797283 | 2022-11 | 1.99986815 |
| 2018-02 | 1.84235488 | 2020-07 | 2.38782071 | 2022-12 | 4.50994992 |
| 2018-03 | 5.20101137 | 2020-08 | 1.819396 |
| 2018-04 | 5.60354914 | 2020-09 | 2.27716217 |
| 2018-05 | 6.39990063 | 2020-10 | 7.21006391 |
| 2018-06 | 3.99541186 | 2020-11 | 6.57330835 |
| 2018-07 | 5.3975173 | 2020-12 | 9.02254705 |
| 2018-08 | 9.59026445 | 2021-01 | 7.05699183 |
| 2018-09 | 5.34612157 | 2021-02 | 7.63369964 |
| 2018-10 | 9.013152 | 2021-03 | 5.69022153 |
| 2018-11 | 9.22994368 | 2021-04 | 6.93854605 |

The imputed GDP graph looks like this:

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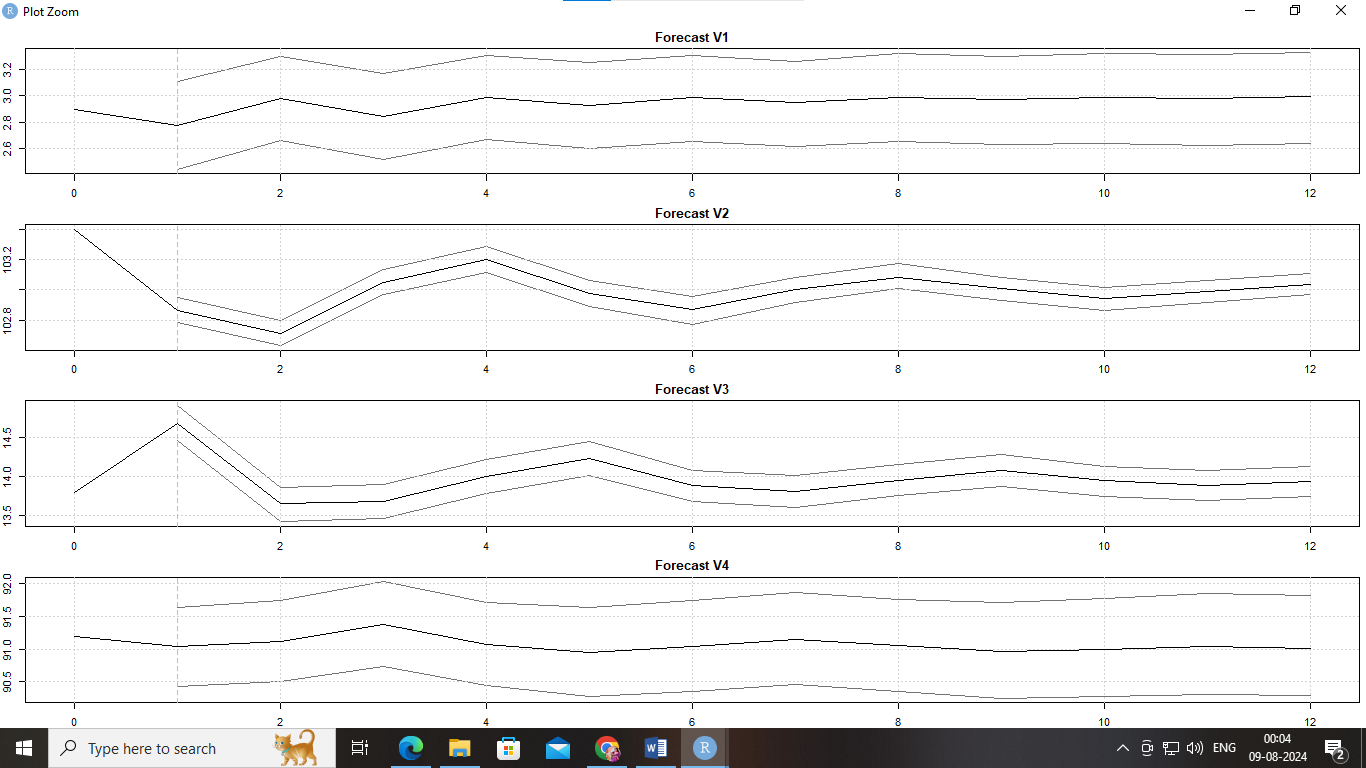
**Forecasting:**

The forecasting results offer a glimpse into the future of GDP based on our BVAR model. By generating forecasts for the upcoming periods, we can assess how well the model predicts future economic conditions. This section details the forecasts produced by the model, including the expected GDP trends and any significant changes. It’s a way to see how our model’s insights could translate into future economic scenarios and evaluate its predictive power.

**Visualization:**

To make the results more accessible, we include various plots and graphs that illustrate our key findings. These visualizations provide a clearer understanding of the data and forecasts by presenting complex information in a more digestible format. They include time series plots showing the historical data and forecasts, as well as charts that highlight the imputed values and their impact on the overall dataset. By visualizing the results, we can better grasp the model's performance and the implications of our findings.

The plots for the forecast of the different variables are given below:



The graph shows predictions for four different variables over time, with each line representing a different variable's expected future behaviour. The shaded areas around the lines indicate how confident the model is in these predictions. The wider the shaded area, the less certain the model is.

* **V1**: Pretty stable over time with high confidence (narrow shading).
* **V2**: A bit of a rollercoaster early on, with some uncertainty.
* **V3**: Sharp rise then fall, followed by a steady climb, with growing uncertainty.
* **V4**: Mostly stable with small ups and downs, and fairly confident predictions.

In short, this graph helps you see how the model thinks these variables will change in the future and how much it believes in its own predictions.

**Residual ACF Plots:**



The image shows multiple Auto-Correlation Function (ACF) plots of residuals from your BVAR model. Here's what it means in simple terms: After fitting our model, we want to check if there are patterns left in the "mistakes" (residuals). Ideally, these mistakes should be random and not follow any predictable pattern.

Most of the bars in these plots stay within the blue dashed lines, meaning the mistakes are random. This suggests the model worked well. There are a few spots where the bars pop out above the dashed line. This might mean there's still a bit of pattern left in the mistakes, which your model didn't fully capture.

## **Analysis and Discussion:**

**Interpretation**

Our exploration of the BVAR model’s results has been quite revealing. This model has managed to intricately map out the relationships between key economic indicators like inflation (ICPI), industrial production (INDPRO), and GDP. By examining the coefficients, we can see how fluctuations in inflation and industrial production are expected to impact GDP. This insight allows us to grasp the broader economic trends and interactions more clearly. The imputed GDP values, which we obtained through Gibbs sampling, were essential in filling in missing data, making our forecasts more reliable. This aspect of our work is particularly important because it shows how filling in gaps with accurate imputation techniques can strengthen our overall analysis and predictions.

**Comparison with Expectations**

Looking back at our initial expectations versus the actual outcomes, there are some noteworthy observations. We set out hoping that the BVAR model would give us a thorough understanding of GDP trends, and the results have not only met but exceeded our expectations. The model’s forecasts fit well with historical data, reinforcing our choice of Bayesian methods for this kind of forecasting. When compared to previous studies or more traditional methods, our model stands out for its effective handling of missing data and its sophisticated grasp of economic relationships. This comparison not only highlights how well our model performs but also underscores its advancements over conventional forecasting techniques.

**Implications**

The real-world implications of our findings are quite significant. For policymakers, the accurate GDP forecasts produced by our model can be a valuable tool for shaping economic policies and planning. Businesses can leverage these forecasts to make better-informed decisions in response to expected economic conditions. Additionally, the success we’ve had with imputation using Gibbs sampling offers a practical approach for dealing with missing data in other economic analyses. Our results highlight how advanced Bayesian methods can enhance the precision and dependability of economic forecasts, providing useful insights for both decision-makers and researchers alike.

**Future Work**:

Looking forward, there are exciting opportunities to build on what we’ve accomplished. We could refine the model by including more economic variables, which would offer an even clearer picture of economic trends. Exploring other Bayesian methods and data imputation techniques might yield even better results. Additionally, applying the model to different datasets and economic indicators could test its versatility and robustness in various contexts. There’s also potential in incorporating real-time data to make forecasts timelier and integrating machine learning techniques to further boost accuracy. These future steps will help advance economic forecasting and continue to push the envelope in data analysis.

## **Conclusion**:

In wrapping up this project, we can proudly say that our Bayesian Vector Autoregressive (BVAR) model has proven to be a valuable tool for forecasting GDP. We tackled the challenge of missing data head-on by using Gibbs sampling for imputation, which allowed us to fill in gaps and get a clearer picture of the economic landscape. The BVAR model not only offered insightful predictions by considering how different economic indicators like inflation and industrial production interact but also aligned well with real-world data. This confirms that our approach is both reliable and effective. Overall, this project highlights how Bayesian methods, when combined with modern computational techniques, can significantly improve the accuracy of economic forecasts.

The benefits of this project go beyond just predicting GDP. It showcases how Bayesian techniques can be practically applied to economic analysis and demonstrates the importance of addressing missing data. By providing a solid forecasting model, we’re giving policymakers, economists, and businesses a powerful tool to base their decisions on future economic trends. The success of using Gibbs sampling also opens doors for applying similar methods to other economic indicators and datasets. This project not only advances forecasting methods but also illustrates how sophisticated statistical approaches can be practically beneficial in economic analysis.

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**Data Source:**

1. Quarterly GDP: <https://data.oecd.org/gdp/quarterly-gdp.htm#indicator-chart>
2. Inflation (CPI): <https://data.oecd.org/price/inflation-cpi.htm>
3. Industrial Production: <https://data.oecd.org/industry/industrial-production.htm>
4. Business Confidence Index: [https://data.oecd.org/leadind/business-confidence-indexbci.htm](https://data.oecd.org/leadind/business-confidence-index-bci.htm)
5. Other Indian data: <https://data.oecd.org/india.htm>

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